Let $P = (-5, -3, 2)$, $Q = (-6, 1, 3)$ and $R = (-1, -7, 4)$. ALL ITEMS WORTH 1 POINT UNLESS OTHERWISE NOTED SCORE: _____ / 13 PTS

[a] Find the area of triangle $\Delta PQR$.

$\overrightarrow{PQ} = \langle -1, 4, 1 \rangle$

$\overrightarrow{PR} = \langle 4, -4, 2 \rangle$

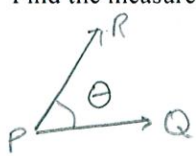
$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 4 & 1 \\ 4 & -4 & 2 \end{vmatrix} = 8\vec{i} + 4\vec{j} + 4\vec{k} - (-4\vec{i} - 2\vec{j} + 16\vec{k}) = 12\vec{i} + 6\vec{j} - 12\vec{k}$$ 3

$$\text{AREA} = \frac{1}{2}\|\langle 12, 6, -12 \rangle\| = \frac{1}{2} \cdot 6 \|\langle 2, 1, -2 \rangle\| = 3\sqrt{4+1+4} = 9$$

[b] Find the vector projection of $\overrightarrow{PQ}$ onto $\overrightarrow{PR}$.

$$\frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\overrightarrow{PR} \cdot \overrightarrow{PR}} \overrightarrow{PR} = \frac{-4-16+2}{16+16+4}\langle 4, -4, 2 \rangle = \frac{\cancel{-18}}{\cancel{36}_2}\langle 4, -4, 2 \rangle = \langle -2, 2, -1 \rangle$$

[c] Find the measure of angle $\angle RPQ$.




$$\theta = \cos^{-1}\frac{-4-16+2}{\sqrt{1+16+1}\sqrt{16+16+4}} = \cos^{-1}\frac{-18}{\sqrt{18}\sqrt{36}} = \cos^{-1}\frac{\cancel{-18}}{\cancel{18}\sqrt{2}}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$$= \frac{3\pi}{4} \text{ or } 135°$$

[d] Find a unit vector perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

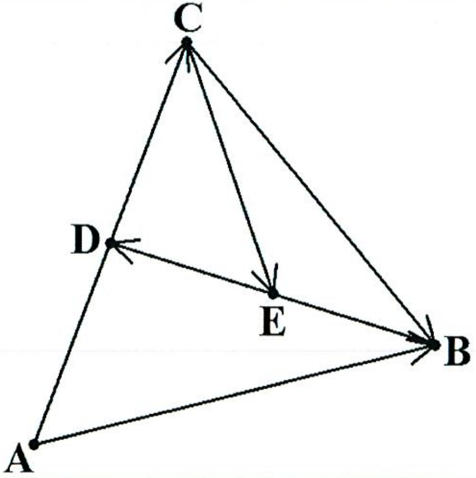
$\overrightarrow{PQ} \times \overrightarrow{PR} \perp \overrightarrow{PQ}, \overrightarrow{PR}$

$$\frac{1}{6\sqrt{4+1+4}}\langle 12, 6, -12 \rangle = \frac{1}{18}\langle 12, 6, -12 \rangle = \left\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right\rangle$$

$\frac{1}{2}$

In the diagram on the right, $D$ is the midpoint of $\overline{AC}$, and $E$ is the midpoint of $\overline{BD}$. SCORE: _____ / 5 PTS

Let $\vec{w} = \overrightarrow{AB}$ and $\vec{z} = \overrightarrow{AC}$.




[a] Find a simplified expression for vector $\overrightarrow{CB}$ in terms of $\vec{w}$ and $\vec{z}$.

$$\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB} = -\vec{z} + \vec{w} = \vec{w} - \vec{z}$$

1

[b] Find a simplified expression for vector $\overrightarrow{BD}$ in terms of $\vec{w}$ and $\vec{z}$.

$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = \overrightarrow{BA} + \tfrac{1}{2}\overrightarrow{AC} = -\vec{w} + \tfrac{1}{2}\vec{z}$$

$$= \tfrac{1}{2}\vec{z} - \vec{w}$$

2

[c] Find a simplified expression for vector $\overrightarrow{CE}$ in terms of $\vec{w}$ and $\vec{z}$.

$$\overrightarrow{CE} = \overrightarrow{CB} + \overrightarrow{BE} = \overrightarrow{CB} + \tfrac{1}{2}\overrightarrow{BD}$$

$$= \vec{w} - \vec{z} + \tfrac{1}{2}\left(\tfrac{1}{2}\vec{z} - \vec{w}\right) = \tfrac{1}{2}\vec{w} - \tfrac{3}{4}\vec{z}$$

2

Let $\vec{q} = <-2, -6, 3>$ and $\vec{r} = <1, -2, -2>$, and let $\vec{p}$ be a unit vector such that the angle between $\vec{p}$ and $\vec{q}$ is $45^\circ$, and the angle between $\vec{p}$ and $\vec{r}$ is $30^\circ$. **SCORE: _____ / 5 PTS**

Find $\vec{p} \times (\vec{q} \times \vec{r})$ using the properties of the various vector operations, and without finding $\vec{p}$.

$$\vec{p} \times (\vec{q} \times \vec{r}) = \underline{(\vec{p} \cdot \vec{r})\vec{q} - (\vec{p} \cdot \vec{q})\vec{r}} \quad 1$$

$$= \underline{(\|\vec{p}\| \|\vec{r}\| \cos 30^\circ)}\vec{q} - \underline{(\|\vec{p}\| \|\vec{q}\| \cos 45^\circ)}\vec{r} \quad \tfrac{1}{2} \quad \tfrac{1}{2}$$

$$= (\sqrt{1+4+4})\left(\tfrac{\sqrt{3}}{2}\right)\langle -2, -6, 3\rangle - (\sqrt{4+36+9})\left(\tfrac{\sqrt{2}}{2}\right)\langle 1, -2, -2\rangle$$

$$= \underline{\tfrac{3\sqrt{3}}{2}}\langle -2, -6, 3\rangle - \underline{\tfrac{7\sqrt{2}}{2}}\langle 1, -2, -2\rangle \quad 1 \quad 1$$

$$= \underline{\left\langle -3\sqrt{3} - \tfrac{7\sqrt{2}}{2}, -9\sqrt{3} + 7\sqrt{2}, \tfrac{9\sqrt{3}}{2} + 7\sqrt{2}\right\rangle} \quad 1$$

$\vec{p}$ IS A UNIT VECTOR $\rightarrow$ $\|\vec{p}\| = 1$

Let $\ell$ be the line passing through $P = (-1, -2, 0)$ and $Q = (-3, -1, -1)$. **SCORE: _____ / 7 PTS**

Let $\wp$ be the plane that is parallel to both $\ell$ and the vector $\vec{m} = 2\vec{j} + \vec{k}$, and also passes through the point $R = (4, 3, -5)$.

[a] Find the symmetric equation of $\ell$.

$\overrightarrow{PQ} = \langle -2, 1, -1 \rangle \longrightarrow$ USE DIRECTION VECTOR $\langle 2, -1, 1 \rangle$

USING POINT P $\quad \underbrace{\frac{x+1}{2}}_{1} = \underbrace{-(y+2)}_{1} = \underbrace{z}_{1}$

[b] Find the point-normal form of the equation of $\wp$.

NORMAL VECTOR $\perp$ $\langle 2, -1, 1 \rangle$ AND $\langle 0, 2, 1 \rangle$

$$\langle 2, -1, 1 \rangle \times \langle 0, 2, 1 \rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = -\vec{i} - (2\vec{i} + 2\vec{j}) + 4\vec{k} = \underbrace{-3\vec{i} - 2\vec{j} + 4\vec{k}}$$

USE NORMAL VECTOR $\langle 3, 2, -4 \rangle$

2 IF ALL COMPONENTS CORRECT

1 IF 2 COMPONENTS CORRECT

$$\underbrace{3(x-4)}_{\frac{1}{2}} \underbrace{+2(y-3)}_{\frac{1}{2}} \underbrace{-4(z+5)}_{\frac{1}{2}} \underbrace{=0}_{\frac{1}{2}}$$